Geometric quantum gates with superconducting qubits

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We suggest a scheme to implement a universal set of non-Abelian geometric transformations for a single logical qubit composed of three superconducting transmon qubits coupled to a single cavity. The scheme utilizes an adiabatic evolution in a rotating frame induced by the effective tripod Hamiltonian which is achieved by longitudinal driving of the transmons. The proposal is experimentally feasible with the current state of the art and could serve as a first proof of principle for geometric quantum computing.

PACS numbers:

I. INTRODUCTION

Superconducting qubits, also known as artificial atoms, have emerged as a promising candidate to achieve quantum computing.[1] The properties of these nanoscale systems can be designed to a large extent, and systems have been found were many logical operations can be performed within the decoherence time.[2,3] Superconducting qubits can be coupled via thin-film microwave cavities[4,5] to allow for two-qubit gates and ultimately for universal quantum computing. Furthermore, they have the intrinsic scalability of condensed matter systems and the high-precision measurement features of quantum optical systems.

Error correction theory predicts that fault tolerant quantum computing requires of the order of 10^4 quantum operations with only a single error on average.[6,7] Contrary to classical computation, where gates and errors are discrete, in quantum computation many small errors can accumulate to an eventual bit or phase flip. Therefore, enormous accuracy for single gates is required. Typically, the control parameters cannot be controlled to such precision and especially the exact timing of control pulses remains challenging. As a possible solution, holonomic quantum computing was proposed.[8] In this case, the unitary transformations depend only on the path which the control parameters trace in parameter space, but not on their timing. Furthermore, random rapidly fluctuating deviations of the actual path to the desired one cancel to the first order.[9,10] Thus, the precision of holonomic quantum gates can possibly be considerably higher that the precision of the control parameters itself.

Abelian holonomies, referred to as geometric phases or Berry phases, have been observed in a wide variety of systems including superconducting qubits.[11,12] The situation is quite different for non-Abelian holonomies necessary for universal geometric quantum computing. Despite several theoretical proposals[13–18], no such adiabatic transformation has been experimentally observed in superconducting qubits, nor in any other systems. Here, we present a scheme for the implementation of a non-Abelian holonomy which is feasible with the devices and methods

used in current experiments on transmon qubits.[2]

Our method is based on the much studied tripod Hamiltonian

$$\hat{H} = \hbar \sum_{j=1}^{3}(\Omega_i|0\rangle\langle i| + \text{h.c.}) \hat{=} \hbar \begin{pmatrix} 0 & \Omega_1 & \Omega_2 & \Omega_3 \\ \Omega_1^* & 0 & 0 & 0 \\ \Omega_2^* & 0 & 0 & 0 \\ \Omega_3^* & 0 & 0 & 0 \end{pmatrix} \quad (1)$$

where the $\Omega_i(t)$ are the control parameters (usually referred to as Rabi frequencies) and the matrix representation is given in the basis $\{|i\rangle\}$, $i = 0, 1, 2, 3$. The first proposal to observe non-Abelian transformations in trapped ions was based on Hamiltonian (1), which is sufficient to implement an arbitray $U(2)$-transformations between the states $|1\rangle$ and $|2\rangle$ used as the logical qubit.[19,20] Similar structures have been recovered in many quantum systems and similar implementations have been proposed[21–24], but yet without experimental verification.

Based on recent experiments[2] we propose a way to implement a universal set of single qubit non-Abelian geometric transformations in a system of three superconducting transmon qubits coupled to the same cavity. Each transmon is composed of two superconducting islands connected by two Josephson junctions, thus forming a superconducting loop.[2,3] The control parameters are the magnetic fluxes through the loops of each transmon, which can be controlled individually allowing us to adiabatically drive the system along a control cycle. With realistic approximations we are able to obtain an effective tripod Hamiltonian in a rotating frame.

The paper is organized as follows. In Sec. II, we introduce the physical setup of the proposed experiment and in Sec. III, we derive the effective tripod Hamiltonian. Section IV is devoted to numerical studies. We conclude the work in Sec. V.

II. PHYSICAL SYSTEM

The superconducting qubits considered here are commonly referred to as transmons.[3,27] Their structure is similar to a charge qubit, but they have a much larger

total capacitance C_Σ such that the ratio of the charging energy $E_C = e^2/(2C_\Sigma)$ over the Josephson energy E_J is much lower than unity. This results in a small charge dispersion of the energy eigenstates, which in turn leads to a significantly reduced sensitivity to charge noise and much longer decoherence times, typically of the order of a few microseconds. On the downside, they have a smaller anharmonicity compared with charge qubits.

Our scheme includes three transmons with frequencies $\varepsilon_i/2\pi$, $i = 1, 2, 3$. The transmons are coupled to a cavity mode with frequency $\omega/2\pi$. The combined system is described by the Tavis-Cummings Hamiltonian[2,3]

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \sum_{i=1}^{3}\left[\frac{1}{2}\hbar\varepsilon_i\hat{\sigma}_z^{(i)} + \hbar g_i\left(\hat{\sigma}_+^{(i)}\hat{a} + \hat{\sigma}_-^{(i)}\hat{a}^\dagger\right)\right] \quad (2)$$

where $g_i/2\pi$ is the transmon–cavity coupling frequency, $\hat{\sigma}^{(i)}$ are the usual Pauli operators for the i-th transmon, and $\hat{a}$ and $\hat{a}^\dagger$ are the bosonic annihilation and creation operators for the cavity mode. To arrive at the above Hamiltonian, we used the rotating wave approximation (RWA), assuming that the coupling strengths g_i are small compared with the excitation energies, which will be the case throughout the paper.

Furthermore, we neglected higher levels of the transmons which is very well justified, because we do not drive the transmons transversally and therefore do not induce excitations to higher energy levels, and we initialize the system in the one-excitation subspace.[25] In this case, the only states involved in the dynamics are $\{|1ggg\rangle, |0egg\rangle, |0geg\rangle, |0gge\rangle\}$, where the first states in the tensor product represents the photon number in the cavity mode and the states $|e\rangle$ and $|g\rangle$ are the excited and ground states of each transmon. The Hamiltonian restricted to this subspace, written in matrix form, reads

$$H = \hbar\begin{pmatrix} 0 & g_1 & g_2 & g_3 \\ g_1 & \Delta_1 & 0 & 0 \\ g_2 & 0 & \Delta_2 & 0 \\ g_3 & 0 & 0 & \Delta_3 \end{pmatrix}, \quad (3)$$

where $\Delta_i = \varepsilon_i - \omega$ is the detuning of the i-th transmon from the cavity (see Fig. 1).

The qubit frequencies ε_i, the detuning Δ_i, and the system–cavity coupling strengths g_i depend on the Josephson energy $E_J^{(i)}(\phi_i)$ and thus on the controllable flux ϕ_i though the i-th qubit. The Josephson energy can be written as $E_J^{(i)}(\phi_i) = E_{J\max}^{(i)}\cos(\pi\phi_i)$ where $E_{J\max}^{(i)}$ is the maximum Josephson energy and ϕ_i is in units of the flux quantum $h/(2e)$. Explicitly, we have[2,3,26]

$$\varepsilon_i(\phi_i) = \sqrt{8E_C^{(i)}E_{J\max}^{(i)}|\cos(\pi\phi_i)|}/\hbar,$$
$$g_i(\phi_i) = k_i[\cos(\pi\phi_i)]^{\frac{1}{4}}, \quad (4)$$

where k_i is a constant depending on the system parameters which can be determined experimentally.

By changing the flux $\phi_i(t)$ through the i-th transmon, we can control the coupling strength $g_i(t)$ as well as the
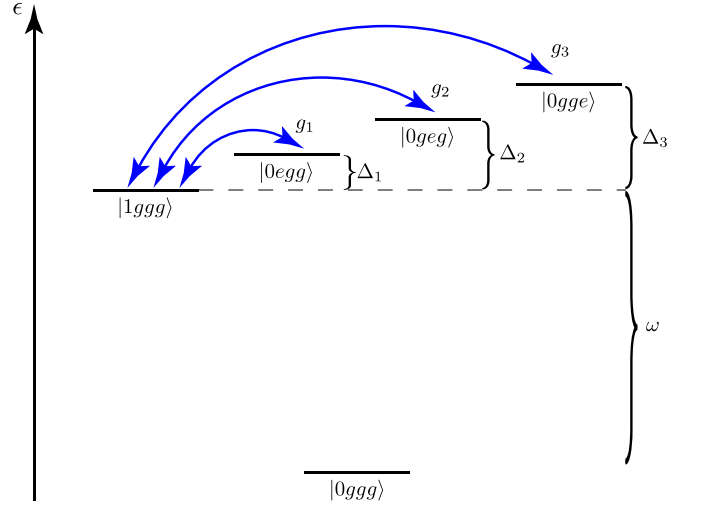

FIG. 1: (Color online) The level structure of the lowest-energy states of the Hamiltonian Eq. (2) are shown. The photon excitation number of the cavity is labeled by $|n\rangle$, while $|g\rangle$ and $|e\rangle$ are the ground and excited states of the three transmons, respectively. The couplings between the one-excitation levels g_i are small compared to the detunings Δ_i.

detuning $\Delta_i(t)$. We separate the the dominant constant contributions [denoted with superscript (0)] which defines the properties of the non-driven system from the small time-dependent ones, which are used to drive the system. We employ the notation

Flux: $\quad \phi_i(t) = \phi_i^{(0)} + \delta\phi_i(t),$
Detuning: $\quad \Delta_i(t) = \Delta_i^{(0)} + \delta\Delta_i(t),$
Coupling: $\quad g_i(t) = g_i^{(0)} + \delta g_i(t),$
Hamiltonian: $H(t) = H^{(0)} + \delta H(t).$

We assume that the flux modulation is small compared to the flux quantum, i.e. $\delta\phi_i(t) \ll 1$, and use a first-order expansion in $\delta\phi_i$ to obtain the time-dependent quantities. Because the cavity frequency is independent of the flux, we have $\delta\Delta_i = \delta\varepsilon_i$. Together with $g_i(\phi) \propto \sqrt{\varepsilon_i(\phi)}$ we obtain a useful relation between the coupling and detuning variations

$$\frac{\delta g_i(\delta\phi_i)}{g_i^{(0)}(\phi_i^{(0)})} = \frac{\delta\Delta_i(\delta\phi_i)}{2\varepsilon_i^{(0)}(\phi_i^{(0)})}, \quad (5)$$

which is valid up to first order in $\delta\phi_i$. Since typically $g_i \ll \varepsilon_i$, the driving via the flux has a much smaller effect on the transmon-cavity coupling g_i than on the detuning Δ_i, and therefore results mainly in longitudnal driving. However, the variations in the detuning induce transitions in higher order perturbation theory, which we refer to as indirect coupling. Whether, the direct or the indirect coupling gives the leading contribution to the effective tripod Hamiltonian depends on the ratio $\Delta_i^{(0)}/\varepsilon_i^{(0)}$ and will be studied below in detail.

III. EFFECTIVE TRIPOD HAMILTONIAN

In this section, we show how to obtain an effective tripod Hamiltonian from the Hamiltonian Eq. (3). In particular, the normal tripod approach which solely utilizes the driving of the off-diagonals of the Hamiltonian will not work for our situation, because our control over g_i is rather limited. Nevertheless, we will find that driving the diagonals results in an indirect coupling of the different eigenstates of $H^{(0)}$ which is of the desired tripod form.

To this end, we assume that the time dependent fluxes $\delta\phi_i(t)$ oscillate with the frequencies $\omega_i/2\pi$ and we write

$$\delta\phi_i(t) = F_i(t)\cos[\omega_i t + \varphi_i(t)],$$

$$\begin{aligned}\delta\Delta_i(t) &= L_i(t)\cos[\omega_i t + \varphi_i(t)],\\ \delta g_i(t) &= T_i(t)\cos[\omega_i t + \varphi_i(t)],\end{aligned} \tag{6}$$

where the adiabatically changing amplitudes $L_i(t)$ and $T_i(t)$ are related with the externally controllable magnitude of the flux oscillations $F_i(t)$ by Eq. (4). To realize a universal set of single qubit transformations, also the relative phases $\varphi_i(t)$ of the oscillations need to change adiabatically in time.[20]

Anticipating that $\delta H(t)$ drives transitions between the eigenstates of $H^{(0)}$, we diagonalize $H^{(0)}$. Up to the first order in $g_i^{(0)}/\Delta_i^{(0)}$, the eigenstates of $H^{(0)}$ in the basis $\{|1ggg\rangle, |0egg\rangle, |0geg\rangle, |0gge\rangle\}$ are given by

$$v_0 \approx \begin{pmatrix} 1 \\ -g_1^{(0)}/\Delta_1^{(0)} \\ -g_2^{(0)}/\Delta_2^{(0)} \\ -g_3^{(0)}/\Delta_3^{(0)} \end{pmatrix}, \quad v_1 \approx \begin{pmatrix} g_1^{(0)}/\Delta_1^{(0)} \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 \approx \begin{pmatrix} g_2^{(0)}/\Delta_2^{(0)} \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 \approx \begin{pmatrix} g_3^{(0)}/\Delta_3^{(0)} \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{7}$$

The Hamiltonian $H(t) = H^{(0)} + \delta H(t)$ in this basis assumes the form

$$H^D(t) = \hbar \begin{pmatrix} 0 & \delta g_1(t) - \frac{g_1^{(0)}}{\Delta_1^{(0)}}\delta\Delta_1(t) & \delta g_2(t) - \frac{g_2^{(0)}}{\Delta_2^{(0)}}\delta\Delta_2(t) & \delta g_3(t) - \frac{g_3^{(0)}}{\Delta_3^{(0)}}\delta\Delta_3(t) \\ \delta g_1(t) - \frac{g_1^{(0)}}{\Delta_1^{(0)}}\delta\Delta_1(t) & E_1 + \delta\Delta_1(t) & 0 & 0 \\ \delta g_2(t) - \frac{g_2^{(0)}}{\Delta_2^{(0)}}\delta\Delta_2(t) & 0 & E_2 + \delta\Delta_2(t) & 0 \\ \delta g_3(t) - \frac{g_3^{(0)}}{\Delta_3^{(0)}}\delta\Delta_3(t) & 0 & 0 & E_3 + \delta\Delta_3(t) \end{pmatrix}$$

where the frequencies E_i can be obtained by perturbation theory

$$E_i \approx \Delta_i^{(0)} + \frac{2(g_i^{(0)})^2}{\Delta_i^{(0)}} + \sum_{j\neq i}\frac{(g_j^{(0)})^2}{\Delta_j^{(0)}}. \tag{8}$$

Here, it is clear that $\delta g_i(t)$ and $\delta\Delta_i(t)$ have to oscillate with frequency $\omega_i = E_i$ to induce an effective coupling. Moving into the rotating frame with respect to the diagonal dominant contribution $H^{\mathrm{diag}} = \mathrm{diag}\{0, E_1, E_2, E_3\}$ and using Eq. (6) we obtain

$$H^D(t) = \hbar \begin{pmatrix} 0 & \Omega_1\{1 + e^{-2i[\omega_1 t + \varphi_1]}\} & \Omega_2\{1 + e^{-2i[\omega_2 t + \varphi_2]}\} & \Omega_3\{1 + e^{-2i[\omega_3 t + \varphi_3]}\} \\ \Omega_1^*\{1 + e^{2i[\omega_1 t + \varphi_1]}\} & L_1\cos[\omega_1 t + \varphi_1] & 0 & 0 \\ \Omega_2^*\{1 + e^{2i[\omega_2 t + \varphi_2]}\} & 0 & L_2\cos[\omega_2 t + \varphi_2] & 0 \\ \Omega_3^*\{1 + e^{2i[\omega_3 t + \varphi_3]}\} & 0 & 0 & L_3\cos[\omega_3 t + \varphi_3] \end{pmatrix}. \tag{9}$$

Here, we defined the effective Rabi frequencies

$$\Omega_i(t) = \left(\frac{T_i(t)}{2} - \frac{g_i^{(0)}L_i(t)}{2\Delta_i^{(0)}}\right)e^{i\varphi_i}$$

$$= L_i(t)\left(\frac{g_i^{(0)}}{4\varepsilon_i^{(0)}} - \frac{g_i^{(0)}}{2\Delta_i^{(0)}}\right)e^{i\varphi_i(t)}, \tag{10}$$

where Eq. (5) was used in the second line. In the RWA we can drop the oscillating entries of Eq. (9) and we arrive at the desired tripod Hamiltonian Eq. (1).

For negative detunings, i.e. $\omega > \varepsilon_i^{(0)}$, the direct coupling due to T_i and the indirect coupling due to L_i add up increasing the strength of the effective coupling. Depending on the ratio between the detuning and the energy gap we have two different regimes. If $|\Delta_i^{(0)}| \ll \varepsilon_i^{(0)}$ we are in the small detuning regime and the second contribution dominates. If $|\Delta_i^{(0)}| \gg \varepsilon_i^{(0)}$ we are in the large detuning regime and the first contribution dominates. Theoretically, both regimes yield the tripod form of the effective Hamiltonian. Since the different regimes have different requirements on the experimental setup, which are readily available for small detuning regime, we study this in more detail in below.

We used two approximations in the derivation of the tripod Hamiltonian. Firstly, $g_i^{(0)} \ll \Delta_i^{(0)}$ was needed to derive Eq. (9). Although there exist higher order terms which might seem to destroy the ideal tripod structure, these can all be removed within the RWA. Nevertheless, there are higher order terms resulting in an effective coupling slightly lower than suggested by Eq. (10) and an optimal driving frequency marginally different from Eq. (8). Secondly, the RWA requires $L_i \ll \omega_i \approx \Delta_i^{(0)}$. Both relations limit the effective coupling strength of the indirect coupling, while the direct coupling is limited by $g_i^{(0)} \ll \varepsilon_i^{(0)}$ which was used to write down Eq. (2). To demonstrate a holonomy with current experimental limitations (decoherence times, transmon-cavity couplings, flux driving), one needs to reduce the detuning $\Delta_i^{(0)}$ to the edge of the validity of the above approximations. This is studied in the following section.

IV. RESULTS

In this section, we verify the validity of above analytical results with numerical studies. As an example, we choose to work with a particular implementation of non-Abelian operator proposed in Refs. 19,20 in which the Rabi frequencies Ω_i are real and parameterized as

$$
\begin{aligned}
\Omega_1 &= \Omega \sin\beta \cos\alpha, \\
\Omega_2 &= \Omega \sin\beta \sin\alpha, \\
\Omega_3 &= \Omega \cos\beta.
\end{aligned}
\tag{11}
$$

Accordingly, in the driving fields in Eq. (6) we take $\varphi_i = 0$. We assume Ω to be constant, while the angles α, and β change adiabatically in time.

Let us discuss some general properties of the tripod Hamiltonian in Eq. (1). It has two non-degenerate eigenstates, usually referred to as the bright states, with energies $\hbar\Omega = \pm\hbar\sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2}$ and, more importantly, a degenerate zero-energy subspace $\mathcal{E}(t) = \mathrm{span}\{|D_1(t)\rangle, |D_2(t)\rangle\}$. These so-called dark states are $|D_1(t)\rangle = \cos\beta(\cos\alpha\,|1\rangle + \sin\alpha\,|2\rangle) - \sin\beta\,|3\rangle$ and $|D_2(t)\rangle = \cos\alpha\,|2\rangle - \sin\alpha\,|1\rangle$. The system state $|\psi(0)\rangle$ is initially prepared to be in this subspace, and if the control parameters are changed adiabatically, then the system
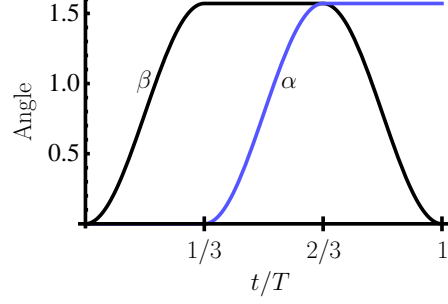

FIG. 2: (Color online) The parameters β (black) and α (blue) as a function of time during the adiabatic control cycle.

will stay in this subspace during the evolution. In particular, for a cyclic Hamiltonian $H(0) = H(T)$ the system will return to the initial subspace $\mathcal{E}(0)$. However, within this subspace the state will undergo a non-trivial $U(2)$ transformation which is the holonomic operator.[28,29]

The evolution in the parameter space begins and ends at the point $(\Omega_1, \Omega_2, \Omega_3) = (0, 0, \Omega)$ and by writing explicitly the dark states using Eq. (11), we obtain that the initial zero-energy subspace is spanned by $\{|1\rangle, |2\rangle\}$. These states are used as basis states of a logical qubit. An adiabatic change of the angles according to

$$
(\alpha(t), \beta(t)): (0, 0) \to (0, \pi/2) \to (\pi/2, \pi/2) \to (\pi/2, 0),
\tag{12}
$$

results up to a phase factor in a holonomic NOT gate $U_{\mathrm{hol}} = |1\rangle\langle 2| - |2\rangle\langle 1|$ for the logical qubit. Note that because of the spherical parameterization, the Hamiltonian is cyclic. For better adiabaticity, we change the angles smoothly using sine functions and constants as shown in Fig. 2.

To implement this loop in our setup, one can control the flux driving amplitudes $F_i(t)$ and hence the longitudinal driving amplitude $L_i(t)$ which is directly related to the Rabi frequencies by Eq. (10). The reference basis is given by Eq. (7) and the initial degenerate subspace corresponds to $|1\rangle \hat{=} v_1$ and $|2\rangle \hat{=} v_2$. In particular, we assume an initial state $|1\rangle$ and, for an ideal transformation, the final state is simply $|\psi_{\mathrm{ideal}}(T)\rangle = |2\rangle$. Thus, the fidelity, defined as $F(t) = |\langle\psi_{\mathrm{ideal}}(t)|\psi(t)\rangle|^2$, after the gate time T is the population of $|2\rangle$. For clarity, we only show the populations of $|1\rangle$ and $|2\rangle$ in the figures below.

We integrate the dynamics of the system using the ideal tripod Hamiltonian in Eq. (1) for different gate times T, as shown in Fig. 3(a) and (b). The fidelity is plotted over the gate time T in Fig. 3(c) and shows the expected approach to unity in the adiabatic limit. The slightly oscillatory behavior observed in Fig. 3(c) is typical for adiabatic gates.[30] However, one should not rely on local maxima of this curve, as their position depends
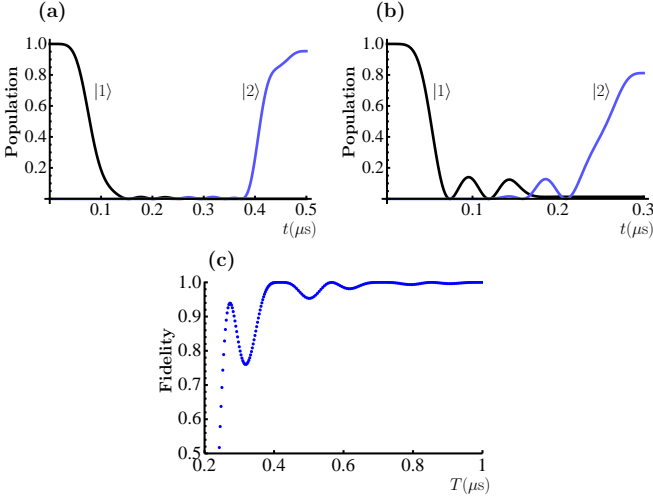
FIG. 3: (Color online) The population of the states $|1\rangle$ (black) and $|2\rangle$ (blue), obtained from the numerical integration of the Schrödinger equation using the effective Hamiltonian in Eq. (1) with $\Omega/2\pi = 10.5$ MHz. (a) The gate time $T = 0.5$ μs shows good population transfer, while (b) for $T = 0.3$ μs the smaller fidelity indicates the invalidity of the adiabatic approximation. Panel (c) shows the gate fidelity as a function of the gate time T.
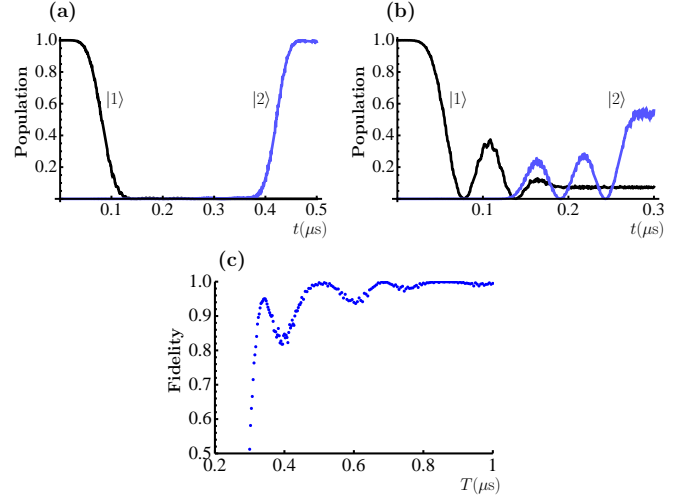

FIG. 4: (Color online) Panels (a) and (b): The population of the states $|1\rangle$ (black) and $|2\rangle$ (blue), obtained from the numerical integration of the Schrödinger equation using the exact Hamiltonian Eq. (3). The parameters are as follows: $\omega/2\pi = 5$ GHz, $g_1^{(0)}/2\pi = 60$ MHz, $g_2^{(0)}/2\pi = -80$ MHz, $g_3^{(0)}/2\pi = 100$ MHz, $\Delta_1^{(0)}/2\pi = -300$ MHz, $\Delta_2^{(0)}/2\pi = -400$ MHz, $\Delta_3^{(0)}/2\pi = -500$ MHz. The longitudinal driving has a restriction of $\max(L_i) = 100$ MHz, resulting in an effective coupling $\Omega/2\pi = 10.5$ MHz. The gate times are $T = 0.5$ μs in panel (a) and $T = 0.3$ μs in panel (b). Panel (c) shows the fidelity as a function of the gate time T.

on the precise value of several experimental parameters. Instead, one should use gate times long enough such that even a local minimum provides a sufficiently good fidelity.

In Fig. 4, we integrate the exact Hamiltonian of Eq. (3). By comparing the results with Fig. 3, we can judge whether approximations such as the rotating wave approximation are satisfied. We use parameters which yield an effective coupling of $\Omega/2\pi = 10.5$ MHz (10 MHz indirect coupling and 0.5 MHz direct coupling). The results follow closely to the ones obtained by the effective tripod Hamiltonian, i.e., a gate time of 0.5 μs results in a fidelity of almost unity, whereas a gate time of 0.3 μs is not enough to justify the adiabatic approximation. The fidelity is plotted as a function of the gate time in Fig. 4(c), which shows much resemblance to the corresponding Fig. 3(c) except for a slight rescaling of the gate time. The reason for this rescaling is that $g_i^{(0)}/\Delta_i^{(0)} = 0.2$ is not small enough to perfectly justify the approximation $g_i^{(0)} \ll \Delta_i^{(0)}$, and therefore the formula Eq. (10) slightly overestimates the effective coupling as described in the previous section.

Because the decoherence time of transmons to date is of the order of a microsecond, one would wish to increase the effective coupling to achieve faster holonomies. This can be done in various ways as shown in Fig. 5(a)–(d), always such that Eq. (10) suggests roughly double the effective coupling compared with Fig. 4(c). One would expect good fidelities in half the gate time compared with Fig. 4(c). The easiest and readily available way is to decrease the detunings Δ_i as shown in Fig. 5(a). However, the gate fidelity is by no means as good as expected from

the effective coupling. The reason is that the conditions $L_i \ll \Delta_i^{(0)}$ and $g_i \ll \Delta_i^{(0)}$ used in the derivation of the effective Hamiltonian tend to get violated for decreasing $\Delta_i^{(0)}$. The situation is slightly better in Fig. 5(b) and (c), where the higher effective coupling is achieved by increasing the transmon–cavity coupling $g_i^{(0)}$ and the driving amplitude L_i, respectively. The only way to increase the effective coupling without affecting the validity of the approximations is to simultaneously increase $g_i^{(0)}$, L_i, and $\Delta_i^{(0)}$, which is shown in Fig. 5(d). However, it may be hard to achieve such high transmon–cavity couplings $g_i^{(0)}$ and driving amplitudes L_i with current setups.

We would like to add a note on experimental feasibility. The parameters used in Fig. 4 are realistic in an existing experimental setup[2,31] and can be achieved [see Eq. (4)], for example, by using transmons with charging energy $E_C = 2\pi\hbar \times 280$ MHz, Josephson energy $E_{J\mathrm{max}} = 2\pi\hbar \times 224$ GHz, fluxes $\phi_1^{(0)} = 0.48426$, $\phi_2^{(0)} = 0.48489$, and $\phi_3^{(0)} = 0.48550$, and variations of the fluxes of up to $\max(\delta\phi_i) = 6.3 \times 10^{-4}$. To verify the geometric transformation, one has to be able to read out the final state of the system. For this purpose, one increases the detunings considerably such that the energy eigenstates v_i are approximate product states of the cavity and the three transmons. Then, it is sufficient to perform state tomography of the first and second transmon because the holonomy is a transformation between
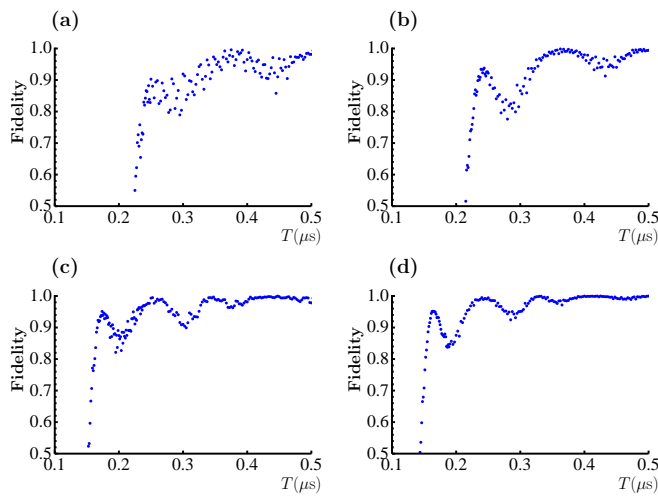
FIG. 5: (Color online) Gate fidelity as a function of gate time T. Parameters are choses as in Fig. 4, but with stronger effective coupling achieved by using half the detuning $\Delta_i^{(0)}$ (a), double the transmon-cavity coupling $g_i^{(0)}$ (b), double the driving L_i (c), and double of all of them (d). This results in the effective couplings $\Omega/2\pi = 20.5$ (a), 21 (b), 21 (c), 22 (d) MHz.

qubits, which could serve as a first step towards geometric quantum computing. Although we did not explicitly take into account the decoherence in our studies, the gate time is within the decoherence time for current experimental setups allowing for proof of principle experiments. The detailed effects of decoherence can be studied along the lines of Refs. 34–36 and will be presented in a future publication. We note that there could be considerable technical improvements in the near future concerning the decoherence time as well as the driving strength, leading to the possibility to carry out extensive small-scale quantum computing. We used the NOT gate as an example to calculate the gate fidelity, but the proposed scheme can be utilized to carry out any single-qubit transformation[20].

v_1 and v_2 only. State tomography has been demonstrated for up to three transmons in Refs. 32,33.

V. CONCLUSION

We proposed an experimental scheme for geometric non-Abelian single-qubit gates with superconducting

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